# COLLAPSE OF THE DUST LAYER ABOVE THE SURFACE OF A BLUNT BODY IN A DUSTY HYPERSONIC FLOW

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Upon flow around a blunt body by a hypersonic flow with a solid impurity the surface of the body is destroyed due to the impact action of the particles. In the course of the destruction the erosion products are carried off into the flow and accumulate in the lateral part of the body directly above the erosion surface. The dust layer formed in this way is effective protection of the surface from further action of the high-velocity particles [1]. A simple computational model of erosional destruction which takes account of the shielding effect has been proposed in [2], in which it was established in particular that in the case of a steady flow a dust layer exists only in a restricted range of variation of the mass concentration of the impurity in the advancing flow. In this connection a model was investigated which includes a description of nonsteady processes in the dust layer. It turned out that the shielding parameter $\Phi$ increases without limit when the mass concentration of the particles exceeds a critical value. The dust layer is strongly compacted and just adheres to the surface of the body, which is interpreted as the initial stage of formation of the coating. It has proven possible within the framework of this model to explain in a simple way the results of the experiments of [3], which are paradoxical at first glance, in which a decrease of the erosion coefficient by 2-3 orders of magnitude was recorded upon multiple collision in comparison with the impact of a single particle.

Erosional processes are investigated in the vicinity of the critical point of an axisymmetric blunt body around which is flowing a dusty hypersonic flow. Along with the usual impact layer of thickness s there exists a thin layer of erosion products which is directly adjacent to the surface of the body, whose thickness $\Delta \ll s$. The equations of mass and momentum transport of a polydisperse mixture are averaged over the thickness of the dust layer in a coordinate system with its y axis along the generatrix of the body. In the single-velocity approximation we have the system of equations (here and later the notation is adopted from [2]):

$$\frac{\partial}{\partial t}(\bar{\rho}\Delta) + \frac{1}{y}\frac{\partial}{\partial y}(y\bar{\rho}v\Delta) = J_e + \rho_c\left(\frac{d\Delta}{dt} - u(\Delta)\right),$$
$$\frac{\partial}{\partial t}(\bar{\rho}v\Delta) + \frac{1}{y}\frac{\partial}{\partial y}(y\bar{\rho}v^2\Delta) + \frac{\partial p}{\partial y}\Delta = \rho_c\left(\frac{d\Delta}{dt} - u(\Delta)\right)v(\Delta), \tag{1}$$

where $\bar{\rho} = \rho_c(1-\alpha) + \rho_e$ and $\alpha$ is the volume content of the erosion products.

In the time-independent case the equations of the model are closed by expressions for the layer thickness $\Delta$, the flow of the erosion products $J_e$, the components of the gas flow velocity on the outer boundary of the layer $u(\Delta)$ and $v(\Delta)$, and the expression for the pressure gradient on the surface of the body. The corresponding expressions obtained from the solution of the "external" problem are of the form [2]

$$\Delta = \Delta_0 e^{-\Phi}, \quad J_e = \rho_{p\infty}u_{p\infty}(1 + E_0 e^{-2\Phi}),$$
$$u(\Delta) = -\frac{2\Delta}{y}\rho_c(1-\alpha)v, \quad v(\Delta) = k_c y, \quad \frac{\partial p}{\partial y} = -\rho_c k_c^2 y. \tag{2}$$

Since the equations of the model contain only three characteristics of the impact layer (the gas density behind the straight section of the shock wave $\rho_c$ and the velocity gradient in the vicinity of the critical point $k_c$ enter explicitly, whereas the effect of the departure distances is exhibited in terms of the parameters $\Delta_0$ and $E_0$), the conclusions obtained within the framework of this model prove to be applicable to the analysis of the erosion of bodies having a blunt shape from a spherical nose to a flat end.

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The boundary conditions for the system (1) are specified on the symmetry axis in the form

$$y = 0: \partial\bar{\rho}/\partial y = 0,\ v = 0. \tag{3}$$

The solution of the problem (1)-(3) in the time-independent case is of the form

$$\rho_e = \rho_c a\Phi e^{\Phi},\ v = \eta k_c y, \tag{4}$$

where

$$\eta = \frac{1 + \sqrt{1 + 3(1 - \alpha + a\Phi e^{\Phi})}}{3(1 - \alpha + a\Phi e^{\Phi})},\quad \eta\Phi = p(1 + E_0 e^{-2\Phi})$$

(for $\alpha = 0$ this solution coincides with that previously obtained in [2]). The parameters $a$ and p are expressed in terms of the relaxation length of the particles $l_p' = \frac{8}{3C_D'}\frac{\rho_s}{\rho_c} r_p$, which is determined from the drag coefficient in the flow of the erosion products $C_D'$ and in terms of other dimensional parameters:

$$a = l_p'/\Delta_0,\quad p = \frac{\rho_{p\infty}}{\rho_c}\frac{u_{p\infty}}{2k_c l_p'}.$$

Thus we obtain a functional relation $\Phi = \Phi(p, a, E_0)$; the erosion coefficient is equal to $E = E_0\exp(-2\Phi)$, by which the shielding effect is explained. Without restricting generality, we further assume $E_0 \ll 1$ and $a \gg 1$. The volume content of particles does not exceed unity, $\alpha < 1$; therefore with $\Phi \sim 1$ we have

$$p = (\Phi e^{-\Phi}/3a)^{1/2} + O(a^{-3/2}).$$

With $\Phi = 1$ p reaches the maximum value ($p^* = (3ae)^{-1/2}$) and then declines as $\Phi$ increases. It follows from this that the function $\Phi(p)$ branches in the vicinity of the point $p = p^*$ and that one should expect critical phenomena here.

We shall consider those perturbations of the parameters of the dust layer for which the structure of the solution (4) is not disrupted. Perturbations with $\Phi = \Phi(t)$, $\eta = \eta(t)$ correspond to this. In addition we shall assume the characteristic Strouhal number $Sh = \dot{\Delta}/u_\infty$ to be so small that closing of the model in the form (2) preserves its meaning. Thus we shall assume $\rho_c$ = const and $k_c$ = const. Discarding the terms in the equations of the system (1) which are small when $\Phi \sim 1$, which correspond to the transport of mass and momentum of the gaseous phase through the boundary of the dust layer, $\rho_e \gg \rho_c$, we have

$$\frac{1}{k_c}\frac{d\Phi}{dt} + 2\Phi\eta = 2p,\ \frac{1}{k_c}\frac{d}{dt}(\Phi\eta) + 3\Phi\eta^2 = \frac{e^{-\Phi}}{a}. \tag{5}$$

The stationary points of the dynamical system (5) which satisfy the conditions $\Phi_0 > 0$, $\eta_0 > 0$ are found from the solution of the system of algebraic equations

$$\eta_0 = (3a\Phi_0 e^{-\Phi_0})^{-1/2},\quad p = (\Phi_0 e^{-\Phi_0}/3a)^{1/2}.$$

We shall investigate the stability of the dynamical system (5) in the vicinity of the stationary points. The roots of the characteristic equation of the corresponding linearized system are

$$\lambda_{1,2} = -\frac{3pk_c}{\Phi_0}\left(1 \pm \sqrt{1 + \frac{2}{3}(\Phi_0 - 1)}\right).$$

Thus the states with $\Phi_0 < 1$ are stable, and the corresponding stationary point $(\Phi_0, \eta_0)$ is a node. When $\Phi_0 > 1$, the states are unstable, and the corresponding point $(\Phi_0, \eta_0)$ is a saddle. We shall consider in what way the system departs from the equilibrium position when $\Phi_0 = 1$. It is convenient to switch from the system (5) to a single equation for the function $\Phi(t)$:

$$\Phi\Phi'' - \frac{3}{2}(\Phi' - 2p)^2 + \frac{2\Phi}{a}e^{-\Phi} = 0,\quad \Phi' = \frac{1}{k_c}\frac{d\Phi}{dt} \tag{6}$$

(it is immediately evident from this equation that the stabilizing effect of a pressure gradient is exhibited only when $\Phi < 1$). We shall set $p = p^* + \varepsilon$, $\varepsilon \ll p^*$, and we shall expand the function $\Phi(t)$ into a series in powers of $\varepsilon$:

$$\Phi = \Phi_0 + \varepsilon\Phi_1 + \varepsilon^2\Phi_2 + \ldots \quad (7)$$

We shall seek the solution of Eq. (6) in the form (7) with the initial data

$$\Phi(0) = 1,\ \Phi'(0) = 2\varepsilon. \quad (8)$$

Substituting the series (7) into Eqs. (6) and equating terms of like powers of $\varepsilon$, we obtain the chain of equations:

$$\Phi_1'' + 6p^*\Phi_1' - 12p^* = 0,$$
$$\Phi_2'' + 6p^*\Phi_2' - 6 = 3(p^*)^2\Phi_1 - 6\Phi_1' + \frac{2}{3}(\Phi_1')^2 - \Phi_1'\Phi_1'',$$
$$\ldots$$

The solution of the problem (6)-(8) is of the form

$$\Phi(t) = 1 + 2\varepsilon k_c t + \frac{\varepsilon^2}{36p^*}\left[18(p^*k_ct)^2 - 6p^*k_ct + 1 - e^{-6p^*k_ct}\right]$$

to within an accuracy of terms $\sim\varepsilon^3$. Thus in the region of small subcriticality the shielding parameter increases with time, and the system leaves the equilibrium position. When $\Phi \gg 1$, one can discard in Eq. (6) the term corresponding to a pressure gradient. After this it is reduced to a first-order equation which is not solved for the derivative:

$$\Phi = C(2p - \Phi')^{2/3}\exp\left(\frac{4p/3}{2p - \Phi'}\right),\quad C = \text{const}.$$

The solution of this equation has the following parametric representation:

$$t = t_0 + \frac{2C}{3k_c}\int_{\xi_0}^{\xi}(2p - \xi)^{-4/3}\exp\left(\frac{4p/3}{2p - \xi}\right)d\xi,$$
$$\Phi = C(2p - \xi)^{2/3}\exp\left(\frac{4p/3}{2p - \xi}\right),\quad \xi < 2p.$$

It follows from this that the shielding parameter increases monotonically with time; $\Phi \to 2pk_ct$ when $t \to \infty$. Due to this the thickness of the dust layer decreases, and the density of the discrete phase in the layer increases with time. Evidently, this process cannot last indefinitely long and concludes when $\rho_e$ reaches some limiting value, after which a coating of densely compacted particles can form in the lateral part of the body. Coatings which form upon collapse of the dust layer were actually recorded in experiments, a detailed description of which can be found in [4]. However, the question of the role of the criterion p* in the coating formation processes remains open.

A possible dependence of the velocity recovery coefficient upon impact $\lambda$ on the collision parameters was not taken into account in the analysis performed above. According to [2], $\Delta = \tau\lambda u_p$. If $\lambda = \lambda_0 u_p^{-\gamma}$ in some range of collision velocities, the shielding parameter above which stability loss of the dust layer occurs is $\Phi_m = (1 - \gamma)^{-1}$. Thus collapse will not be observed in the transition region from inelastic to elastic impact, in which $\gamma \gtrsim 1$.

Special calculations were made to estimate the effect of the erosion parameter $E_0$ on the critical values of particle discharge, which showed that this effect is inappreciable. Thus the critical discharge p* decreases by approximately 27% for $a = 20$ upon a variation of $E_0$ by two orders of magnitude.

The erosion coefficient E decreases strongly with an increase in the shielding parameter; therefore one should expect in experiments on erosion in hypersonic flows with a dustiness above the critical level an appreciable decrease in the destruction observed in comparison with the theoretical estimates based on data for a single collision. The dust layer which is formed on the lateral part of the body can shield the surface not only partially but completely from the action of high-velocity particles. The resulting amount of erosion damage of the surface is determined by the initial period of formation of the dust layer with a characteristic time for the process of $\Delta t_{er} \sim 1/2pk_c$.

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# CALCULATION OF THE VIRTUAL MASS OF SPHERICAL PARTICLES IN A DISPERSED MEDIUM


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One of the central problems in the mechanics of multiphase dispersed media is the problem of determining the interphase interaction. This problem is most simply solved by using relations which are valid for a single inclusion, moving in an unbounded carrying medium. This approach, however, does not take into account the effect of the inclusions on one another through the carrying medium, which can lead to considerable errors in the determination of the interphase interaction [1, 2].

The effect of inclusions on one another is easy to take into account within the framework of the cellular approach, which is analyzed in [1]. The method of cells is applicable to the study of media with a regular structure. Dispersed media, as shown in [1], have different microstructures under different conditions: a regular structure, when the distance between neighboring inclusions is the same; a chaotic structure, when the inclusions are distributed randomly, and others.

In the general case, the average interphase interaction force is found by averaging the interphase interaction force over all positions of the inclusions. However, the distribution function of the positions of the inclusions depends on the interphase interaction force. Thus, to determine the average interphase interaction, it is necessary to solve a very complicated problem.

Except for the rare exception [3], in solving this problem it is assumed that a dispersed medium has either a regular or chaotic structure. However, even after this assumption is made, it is difficult to determine the average interphase interaction, since it is difficult to determine the interphase interaction for a specific distribution of inclusions. For this reason, many authors use different simplifying assumptions in calculating the average interphase interaction [4-6]; in addition, within the framework of their approach, it is impossible to estimate the error introduced by these assumptions. The results obtained using the exact solution of the problem of interaction of several inclusions in the carrier medium are more reliable [7, 8].

In this paper, we examine the problem of the motion of spherical inclusions in an ideal carrier medium. We describe the technique for calculating the average characteristics of the interaction of inclusions with the carrier medium. This technique is used to calculate the virtual mass of spherical inclusions in the dispersed medium.


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